$1.62$ An Infinite Series.

It is not a difficult proposition to apply residue theory (from complex analysis) to evaluate the infinite sum

$$\sum_{n=0}^{\infty} \frac{1}{n^2 + n + 1} = \frac{\pi}{\sqrt{3}} \tanh \frac{\sqrt{3}}{2} \pi.$$ 

Using only this information to your advantage, evaluate the sum

$$\sum_{n=1}^{\infty} \frac{1}{n^2 - n + 1}.$$